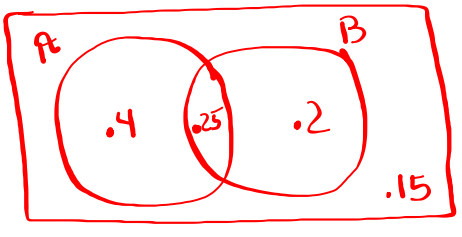
Math 110
Winter 2021
Lecture 11

Class QZ 6

Given $P(A) = .65$ $P(B) = .45$ $P(A \text{ and } B) = .25$

1) Venn Diagram



Exam II is Next Monday

2) $P(\bar{A})$

$= 1 - .65 = .35$

3) $P(A \text{ or } B)$

$= .4 + .25 + .2 = .85$

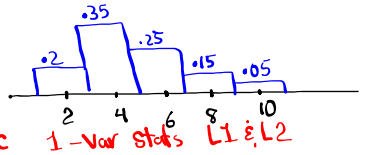
Consider the following chart

x	$P(x)$
2	.20
4	.35
6	.25
8	.15
10	.05

1) Find $P(x=2)$

$= 1 - [.35 + .25 + .15 + .05] = 1 - .8 = .2$

2) Draw Prob. dist. histogram



3) Find μ & σ

$x \to L1$ STAT CALC 1-Var Stats L1 & L2

$P(x) \to L2$ $\mu = 5$ $\sigma = 2.236$

4) Find σ^2 in reduced fraction $\sigma^2 = 5$

VARS 5: 4: x^2 Math 1: Enter

5) Round μ & σ to 1-decimal, then find usual Range (95% Range)

$\mu = 5.0$

$\sigma = 2.2$

$\mu \pm 2\sigma$

$= 5.0 \pm 2(2.2) = 5 \pm 4.4 \Rightarrow$ .6 to 9.4

Application

Expected Value $= \mu = \bar{x}$

I sold 50 tickets for $10 each.

One ticket is randomly selected, the owner get a TI-84, worth $125. Find expected value per ticket sold.

$Net	P($Net)
$10 − 125	$\frac{1}{50}$
$10 − 0	$\frac{49}{50}$

$Net → L1, P($Net) → L2

STAT CALC 1-Var Stats

L1 & L2

Expected Value $= \mu = \bar{x} = 7.5$

$7.50 / ticket

Before going to surgery, you buy insurance for $500. IF you don't make it, your family gets $100,000 in benefit.

Prob. that you don't make it is .5%. what is the expected value per policy sold by insurance company?

$Net	P($Net)
$500 - 100,000	.5% = .005
$500 - 0	99.5% = .995

$Net → L1

P($Net) → L2

E.V. = $\mu = \bar{x} =$ **0**

A box contains 2 Quartes and 13 Nickels.

Let's shake it to get 2 coins.

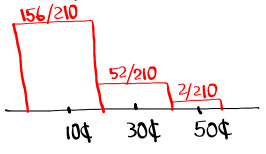
QQ → 50¢ P(collect 50¢) = $\frac{2}{15} \cdot \frac{1}{14} = \frac{2}{210}$

QN, NQ → 30¢ P(30¢) = $1 - \left[\frac{2}{210} + \frac{156}{210}\right] = 1 - \frac{158}{210} = \frac{52}{210}$

NN → 10¢ P(collect 10¢) = $\frac{13}{15} \cdot \frac{12}{14} = \frac{156}{210}$

Collect ¢	P(collect ¢)
10¢	$\frac{156}{210}$
30¢	$\frac{52}{210}$
50¢	$\frac{2}{210}$

Prob. Dist. histogram



Collect ¢ → L1

P(collect ¢) → L2

Sind

$\mu = 15.333$

$\sigma = 9.265$

Round μ & σ to a whole #, Sind

σ^2 (reduced fraction) = $\frac{5408}{63}$

68% Range ⇒ $\mu \pm \sigma$

$\mu = 15$

$\sigma = 9$

$= 15 \pm 9$

⇒ **6 to 24**

Binomial Prob. Dist.

1) n independent events

2) Each event has only two outcomes.
   - Success
   - Failure

3) $P(\text{Success}) = p$, $P(\text{Failure}) = q$
   $p + q = 1$, $q = 1 - p$
   p & q remain uncharged for all n events

4) x is # of Successes

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

Consider a binomial prob. dist with $n = 20$,

$p = .6$

Find $P(x = 15) = {}_{20}C_{15} \cdot (.6)^{15} \cdot (.4)^{5}$

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

20 MATH PRB $_nC_r$ 15 * .6 ^ 15 * .4 ^ 5 Enter

$= .075$

You are taking a quiz, 40 questions, True/False only, and making random guesses.
What is the prob. that you correctly guess 25 correct answers?

$n=40$, $p=.5$ $q=.5$

$P(x=25) = {}_{40}C_{25} \cdot (.5)^{25} \cdot (.5)^{15} =$ **.037**

2nd | VARS | ↓↓... | binomPdf

Menu
Trials: 40
P: .5
x Value: 25
Paste | Enter | Enter

No Menu
binompdf(40, .5, 25)
n P x
Enter

.037

A loaded coin is tossed 100 times.

$P(\text{Tails}) = .6$

P(land exactly 75 Tails)

$= P(x=75) =$ binompdf(100, .6, 75) $=$ **6.3×10^{-4}**
n P x

P(at most 50 tails) $= P(x \leq 50)$

= binomcdf(100, .6, 50)
0, 1, 2, 3, ..., 48, 49, 50

= **.027**

UPS says prob. that any packages arrives on time or early is 90%..

Let's randomly select 80 packages

1) P(exactly 70 arrive ontime or early)

$P(x=70) = \text{binompdf}(80, .9, 70) = \boxed{.103}$

2) P(fewer than 75 are ontime or early)

$P(x < 75) = P(x \leq 74) = \text{binomcdf}(80, .9, 74)$

$= \boxed{.823}$

You are taking an exam with 50 multiple-choice questions.

Each question has 4 choices with only one correct choice. You are making random guesses.

$n = 50 \quad p = \frac{1}{4} = .25 \quad q = \frac{3}{4} = .75$

P(exactly 15 correct answers) $= P(x = 15)$

$= \text{binompdf}(50, .25, 15)$

$= \boxed{.089}$

P(fewer than 20 correct answers) =

$P(x < 20) = P(x \leq 19) = \text{binomcdf}(50, .25, 19)$

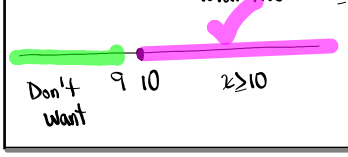
$= \boxed{.986}$

P(at least 10 correct answers) =

$P(x \geq 10) = 1 - P(x \leq 9)$

$= 1 - \text{binomcdf}(50, .25, 9)$

$= \boxed{.836}$



100 Newborn babies were randomly Selected,

$n=100$, $p=.5$, $q=.5$

P(exactly 45 boys) $= P(x=45)$

$=$ binompdf(100, .5, 45)

$=$ .048

P(at most 60 girls) $= P(x \le 60)$

$=$ binomcdf(100, .5, 60)

$=$ .982

P(at least 45 boys) $= P(x \ge 45)$

$= 1 - P(x \le 44)$

$= 1 -$ binomcdf(100, .5, 44)

$=$ .864

$P(x=a) =$ binompdf(n, p, a)

$P(x \le a) =$ binomcdf(n, p, a)

$P(x \ge a) = 1 -$ binomcdf$(n, p, a-1)$

$P(a \le x \le b) =$ binomcdf$(n, p, b) -$ binomcdf$(n, p, a-1)$

Reduce by 1

binomial Prob. dist with $n=40$, $p=.8$

$P(30 \le x \le 35) =$ binomcdf(40, .8, 35) $-$ binomcdf(40, .8, 29) $=$ .763

Reduce by 1

Binomial Prob. Dist:

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard Deviation $\sigma = \sqrt{\sigma^2}$

Toss a fair coin 400 times

$n = 400 \quad P = .5 \quad q = .5$

$\mu = np = 400(.5) = \boxed{200}$

$\sigma^2 = npq = 400(.5)(.5) = 100$

$\sigma = \sqrt{\sigma^2} = \sqrt{100} = \boxed{10}$

Usual Range

$\mu \pm 2\sigma$

$= 200 \pm 2(10)$

$= 200 \pm 20$

180 to 220

Usual Range

95% "